Bài 1 (trang 57 SGK Đại số 11):

Viết khai triển theo công thức nhị thức Niu – tơn:

a. $(a + 2b)^5$;

b. $\left(a-\sqrt{2}\right)^6$;

c. $\left(x-\frac{1}{x}\right)^{13}$.

Lời giải chi tiết:

a. $(a+2b)^5$

$= C_5^0.a^5 + C_5^1.a^4.(2b) + C_5^2.a^3.(2b)^2$

$+C_5^3.a^2.(2b)^3 + C_5^4.a.(2b)^4 + C_5^5.(2b)^5$

$= a^5 + 5.a^4.2b + 10.a^3.4b^2$

$+10.a^2.8b^3 + 5.a.16b^4 + 32b^5$

$= a^5 + 10a^4b + 40a^3b^2$

$+80a^2b^3 + 80ab^4 + 32b^5$

b. $\left(a-\sqrt{2}\right)^6$

$= \left[a+\left(-\sqrt{2}\right)\right]^6$

$= C_6^0.a^6 + C_6^1.a^5.\left(-\sqrt{2}\right) + C_6^2.a^4.\left(-\sqrt{2}\right)^2$

$+ C_6^3a^3\left(-\sqrt{2}\right)^3 + C_6^4a^2.\left(-\sqrt{2}\right)^4 +$

$+ C_6^5.a.\left(-\sqrt{2}\right)^5 + C_6^6.\left(-\sqrt{2}\right)^6$

$= a^6 + 6.a^5.\left(-\sqrt{2}\right) + 15.a^4.2 + 20.a^3.\left(-2\sqrt{2}\right)$

$+ 15.a^2.4 + 6.a.\left(-4\sqrt{2}\right) + 8$

$= a^6 - 6\sqrt{2}.a^5 + 30a^4 - 40\sqrt{2}.a^3$

$+60a^2 - 24\sqrt{2}.a + 8$

c. $\left(x-\frac{1}{x}\right)^{13}$

$=\left[x+\left(-x^{-1}\right)\right]^{13}$

$= C_{13}^{0}.x^{13} + C_{13}^{1}.x^{12}.\left(-x^{-1}\right) + C_{13}^{2}.x^{11}.\left(-x^{-1}\right)^{2}$

$+ \ldots + C_{13}^{k}.x^{13-k}.\left(-x^{-1}\right)^{k}$

$+ \ldots + C_{13}^{12}.x.\left(-x^{-1}\right)^{12} + C_{13}^{13}.\left(-x^{-1}\right)^{13}$

$= C_{13}^{0}.x^{13} + C_{13}^{1}.x^{12}.(-1).x^{-1} + C_{13}^{2}.x^{11}.x^{-2}$

$+ \ldots + C_{13}^{k}.x^{13-k}.(-1)^{k}.x^{-k}$

$+ C_{13}^{12}.x.x^{-12} + C_{13}^{13}.(-1).x^{-13}.$

$= C_{13}^{0}.x^{13} - C_{13}^{1}.x^{11} + C_{13}^{2}.x^{9}$

$+ \ldots + (-1)^{k}.C_{13}^{k}.x^{13-2k}$

$+ \ldots + C_{13}^{12}.x^{-11} - C_{13}^{13}.x^{-13}.$

Kiến thức áp dụng:

+ Khai triển nhị thức Niu-tơn

$$(a+b)^{n} = C_{n}^{0}.a^{n} + C_{n}^{1}.a^{n-1}.b + \ldots + C_{n}^{k}.a^{n-k}.b^{k} + \ldots + C_{n}^{n}.a.b^{n-1} + C_{n}^{n}.b^{n}$$